Section 6.2- Tests of significance

EXPERIMENT:

claim: $p_{RED} = 0.25$

$n = 20$

RED	BLUE
I	~~IIII~~ ~~IIII~~ ~~IIII~~ IIII

$\hat{p} = 0.05$

$\hat{p} = 0.30$ $\quad p = 0.25$

$\hat{p} = 0.35$ $\quad p = 0.25$

$\hat{p} = 0.39$

$\hat{p} = 0.43$

4 R

16 B

20

6.2

1. to assess the evidence for/against a claim sample
2. sample to claim
3. prob. of getting our sample if claim is true

1. Hypotheses (claim & alternative)
   $p = 0.60$
   $p < 0.60$
2. Test Statistic (standardizing z-scores)
3. P-value (prob)
4. Conclusion (2 sentences)

Ex:

① Hypotheses

$H_0: \mu = 90$

$H_a: \mu < 90$

$\mu = 90$

$\sigma = 2.2$

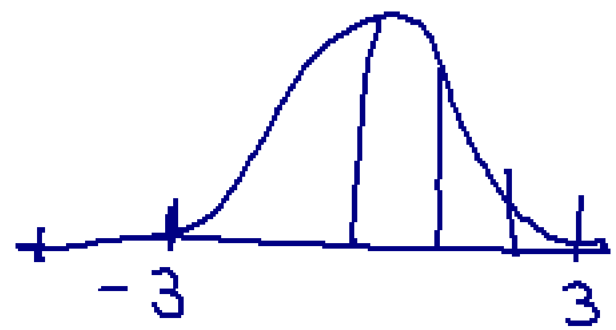
$n = 10$

$\bar{x} = 86.2$

$\alpha = 0.05$

alpha

② Test Statistic



$$z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} = \frac{86.2 - 90}{2.2/\sqrt{10}} = -5.4620$$

③ P-value

$$P(z < -5.462) = 2.3596 \times 10^{-8}$$

4 - We reject H_0 b/c p-value $< \alpha = 0.05$.

- We sufficient evidence that the avg. pitch speed is less than 90 mph.

$\alpha = 0.05 = 5\%$

Warm Up:

1)

State	Check
1. SRS	1. assumed
2. σ known	2. circled $\sigma=$
3. norm. pop. or $n \geq 30$	3. $n=100 \geq 30$

$n=100$

$\sigma=3{,}569$

$\bar{x}=65{,}358$

98% conf

$$\bar{x} \pm z^* \frac{\sigma}{\sqrt{n}} =$$

(64528, 66188)

We are 98% confident that the true average income of Bucks County residents is between \$64,528 and \$66,188.

Warm Up:

2) $\mu = 58$ $\quad n = 25$ $\quad \alpha = 0.01$

$\sigma = 9.1$ $\quad \bar{x} = 61$

$H_0: \mu = 58$

$H_a: \mu > 58$

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \boxed{1.6484}$$

$$P(z > 1.6484) = \boxed{0.0496}$$

STATE	CHECK
① SRS	① circled
② σ known	② circled
③ norm. pop or $n \geq 30$	③ assumed normal

- We fail to reject H_0 because p-value > $\alpha = 0.01$

- We have sufficient evidence that the true average # of pieces of candy in a bag is equal to 58 pieces.

Example #2:

$\mu = 725$

$\sigma = 125$

$\bar{x} = 670$

$n = 40$

$\alpha = 0.01$

$H_0: \mu = 725$

$H_a: \mu < 725$

$$z = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \boxed{-2.7828}$$

$$P(z < -2.7828) = \boxed{0.00269}$$

- We reject Ho b/c p-value < $\alpha = 0.01$.
- We have sufficient evidence that the true average salary for male managers is less than \$725.

Notes on Components of Tests of Significance:

Hypotheses

* always describe... population parameters (μ, p)
* Symbols:

Null Hypothesis

$H_0: \mu =$

- claim
- believed to be true

* write before data collected

Alternative Hypothesis:

H_a:

- suspected to be true

$H_a: \mu \;>,\ <,\ \neq$ ___#___

One sided: $>$, $<$

Two sided: $\neq$

Test Statistic:

GENERIC:

$$\text{test stat} = \frac{\text{statistic} - \text{parameter}}{\left(\begin{array}{c}\text{std. dev.}\\ \text{of stat.}\end{array}\right)}$$

SPECIFIC (to Ch.6, testing means):

prop.

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

$$z = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}}$$

P-Value

**DEFINITION:
- probability of getting our sample or something more extreme if Ho is true.

Calculation:

$P(Z \gtrless \underline{\text{test stat}})$ Ha

* smaller the p-value...
more evidence against our claim

How small is small enough to say the claim isn't true?
α = decision level

GIVEN IN PROBLEM or $\alpha = 0.05$

Common levels: 0.01 0.05 0.10

Conclusion

Two conclusions:

REJECTING Ho:

- We reject Ho b/c p-value < α = ___.
- Suff. evid. that ..... (Ha)

FAILING TO REJECT Ho:

- We fail to reject Ho b/c p-value $\geq \alpha$ =
- Suff. evid. that .... (Ho)

Our conclusion must always be a complete sentences **in terms of** Ho

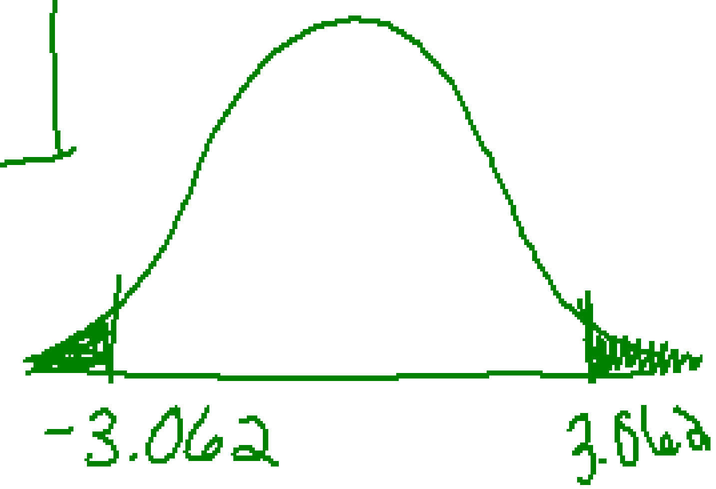
Example 3:

$H_0: \mu = 39.5$

$H_a: \mu \neq 39.5$

$$z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} = \boxed{3.062}$$

$\mu = 39.5$
$n = 150$
$\bar{x} = 42$
$\sigma = 10$
$\alpha = 0.05$



P-VALUE:

$2 \cdot P(z > 3.062)$

$2 \cdot \text{normcdf}(3.062, 1E99, 0, 1)$

$= 0.0022$

– We reject Ho b/c p-value < $\alpha = 0.05$

– Suff. evid. that avg ≠ 39.5 yrs old

** Complete all of worksheet 6.2A